AP CALCULUS	LECTURE NOTES	MR. RECORD
Section Number: **3.1**	**Topics:** Extrema on an Interval	**Day:** 1 of 1

I. Extrema of a Function

In calculus, much effort is devoted to determining the behavior of a function *f* on an interval *I*. We will investigate such questions as

Does f have a maximum value on I? Does it have a minimum value on I?

When is the function increasing? When is it decreasing?

Throughout this chapter, you will use derivatives to answer these questions and more and then apply them to real-life situations.

A function does not have to have a maximum or a minimum on an interval. For instance, in Figure 1 you can see that the function $f(x) = x^2 + 1$ has both a maximum and a minimum on the closed interval $[-1, 2]$

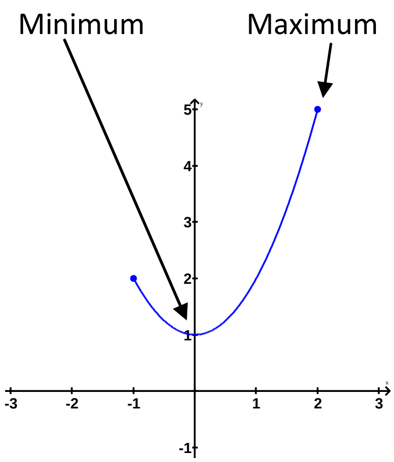


Figure 1

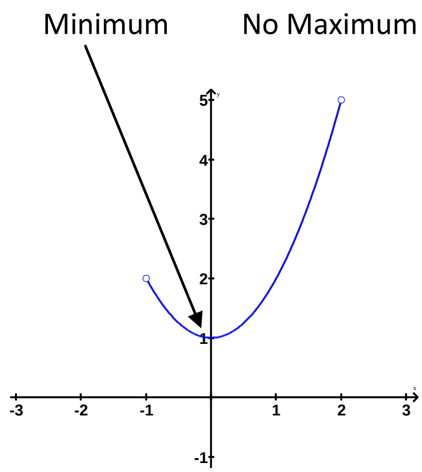


Figure 2

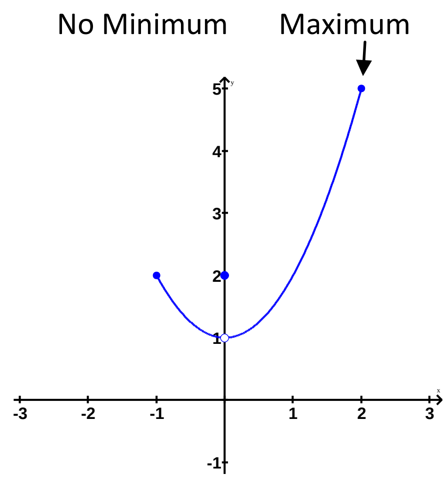
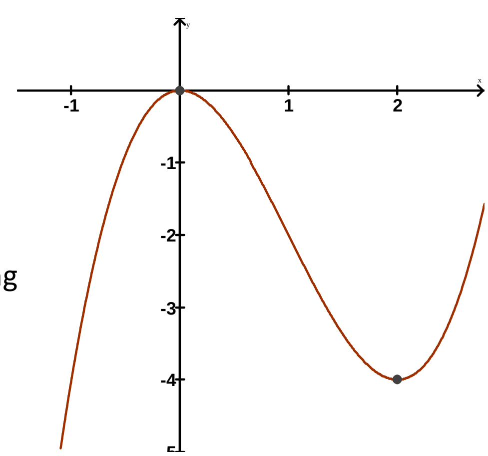


Figure 3

The following Theorem is a result from the situation above:

> **The Extreme Value Theorem**
>
> If *f* is continuous on a closed interval $[a, b]$, then *f* has BOTH a maximum and a minimum on the interval.

II. Relative Extrema and Critical Numbers



The graph of $f(x) = x^3 - 3x^2$ to the right has a **relative minimum** at the point (0, 0) and **relative maximum** at the point (2, -4)

To make this easier for you, think of the relative maximum as being a "peak" of a mountain on the graph and the relative minimum occurring as the "valley" of the graph.
Note that if these "peaks" and "valleys" have smooth, rounded curves about them, then the graph has a horizontal tangent line at those points.

Definition of Relative Extrema

Let f be a function whose second derivative exists on an open interval I.

1. If there is an open interval containing c on which $f(c)$ is a maximum, then $f(c)$ is called a **relative maximum** of f.
2. If there is an open interval containing c on which $f(c)$ is a minimum, then $f(c)$ is called a **relative minimum** of f.

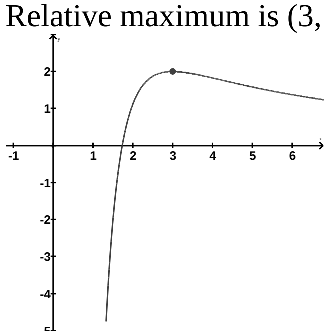
The plural of relative maximum is relative maxima, and the plural of relative minimum is relative minima.

Example 1: The Value of a Derivative at Relative Extrema

Find the value of the derivative at each of the relative extrema shown for each graph.

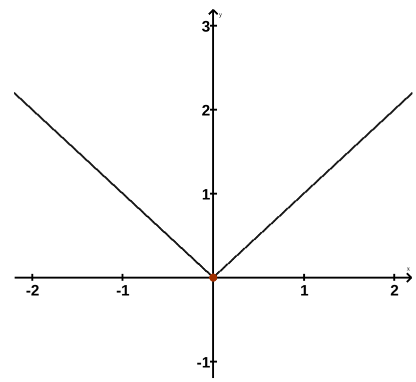
a. $f(x) = \dfrac{9(x^2-3)}{x^3}$

Relative maximum is (3, 2)



b. $f(x) = |x|$

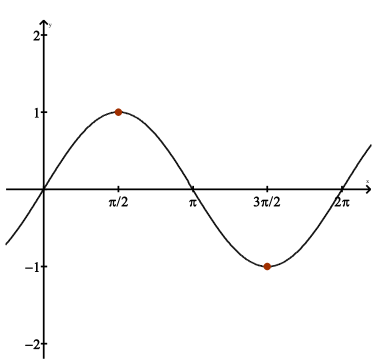
Relative minimum is (0, 0)



c. $f(x) = \sin x$

Relative minimum is $\left(\dfrac{3\pi}{2}, -1\right)$

Relative maximum is $\left(\dfrac{\pi}{2}, 1\right)$

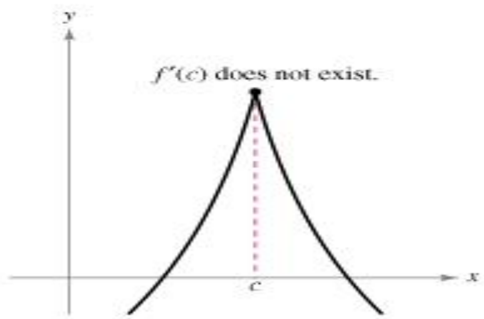


Note that at each relative extrema, the derivative is either zero or does not exist. The *x*-values at these special points are called **critical numbers.**

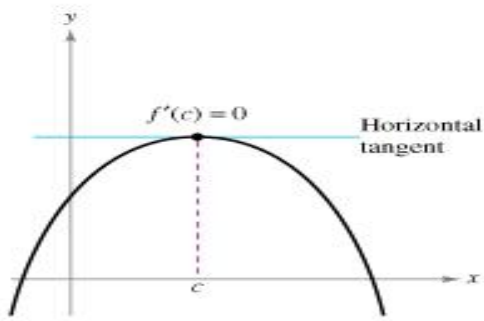
Definition of Critical Numbers

Let *f* be defined at *c*. If *f* ' (c) = 0 or if *f* is not differentiable at *c*, then *c* is a **critical number** of *f*.

Below are two examples of a critical number, *c* of function *f*.



c is a critical number of *f*.



Relative Extrema Occur Only at Critical Numbers

If *f* has a relative minimum or relative maximum at *x*= *c*, then *c* is a critical number of *f*.

III. Finding Extrema on a Closed Interval

Guidelines for Finding Extrema on a Closed Interval

To find extrema of a continuous function *f* on a closed interval $[a,b]$, use the following steps.

1. Find the critical numbers of *f* in (*a*, *b*).
2. Evaluate *f* at each critical number in (*a*, *b*).
3. Evaluate *f* at each endpoint of [*a*, *b*].
4. The least of these values is the minimum. The greatest is the maximum.

Example 2: Finding Extrema on a Closed Interval

a. Find the extrema of $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$.
Use your calculator to sketch the graph and verify.

b. Find the extrema of $f(x) = 2x - 3x^{2/3}$ on the interval $[-1, 3]$.
Use your calculator to sketch the graph and verify.

c. Find the extrema of $f(x) = 2\sin x - \cos 2x$ on the interval $[0, 2\pi]$.
Use your calculator to sketch the graph and verify.